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PARAMETRIC AND DYNAMIC PROGRAMMING IN FOREST FIRE CONTROL MODELS

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ABSTRACT

The forest fire control models developed by Parks and Jewell [1] [3] assumed one type of suppression force with a particular effectiveness and cost of operations. These models are extended here to the multiple suppression force case.

If N types of forces are dispatched to a fire, they will either arrive simultaneously or in a lagged sequence. Choosing the optimal (lowest cost of burn plus suppression) mix of forces for simultaneous arrival requires the use of a linear programming model that is simultaneously parametric in the right-hand sides and in the objective function. For non-simultaneous arrival, a dynamic programming algorithm, that compared pairs of forces, was developed and solved on an IBM 1620 computer. The lowest envelope of cost curves for all such pairs can be used as a planning tool in initial attack.

PARAMETRIC AND DYNAMIC PROGRAMMING IN FOREST FIRE CONTROL MODELS

I. Introduction

The fire growth model of Parks and Jewell [1] was constructed under the assumption that the suppression force has a singular effectiveness denoted by E. In actual practice, however, suppression forces form a complex organization with different values of E for each component. The extension of Parks' model to the multiple suppression crew case is necessary for its actual employment in initial attack planning.

This paper describes the construction of and application of some decision rules concerning the choice of forces sent to a fire given the essential fire characteristics, the values of E, and the attack time for each group of suppression forces. The decision criterion used here is minimization of total costs incurred after discovery of a fire. An alternative criterion of decision, commonly used by the U. S. Forest Service, is acreage control. Therefore, wherever possible in the mathematical solutions, the optimal solutions for all values of acres burned after detection will be included with the overall optimal solution.

The assumptions used here are basically those that appear in Reference [1]. The most important of these is,

$$\frac{dy}{dt} = G_d + Ht$$

where dy/dt is the growth rate of the fire in acres per hour, G_d is the observed growth rate at detection. H is the acceleration in acres per hr^2, and t is any time after detection. After initial attack, the growth rate is described by,

$$\frac{dy}{dt} = G_a - (EX - H)(t - T_a)$$

where X is the number of men sent, T_a is the time interval between detection and attack, E is the effectiveness of the men, and G_a is the growth rate at the time of attack. For multiple forces two problems arise: 1) measurement of the various E values for each type of force, and 2) description of the growth rate in terms of nonsingular attack times. Measurement of E values for hand crews shows a clear relationship of E with the basic line building rates. A more detailed analysis of this problem is treated by McMasters [4]. Let us concern ourselves with the second problem.

In general, any group of forces will not arrive at the fire simultaneously. The decision maker must decide how many of each type of force to send given their effectiveness and inter-arrival times. As a simplifying assumption, however, consider first the case of simultaneous arrival.

II. MODEL 1: Simultaneous Arrival

Simultaneous arrival is possible both in actual practice and as an assumption when the inter-arrival times are small compared to the detection-arrival interval. Suppose a forest or region has N types of suppression forces available. Let X_i = the amount of the i^{th} type of force sent to the fire

E_i = effectiveness of the i^{th} force in acres per man hr^2

Y_c = total acres burned at control

Y_a = total acres burned at initial attack

G_a = growth rate at initial attack

The fire is controlled when

$$\frac{dy}{dt} = 0;$$

This can occur if $\sum_i E_i X_i \geq H$,

and therefore by extension of Parks' equations,

$$(1) \qquad Y_c - Y_a = \frac{G_a^2}{2(\sum_i E_i X_i - \dot{H})}$$

and T_c , the control time, is given by,

$$(2) \qquad T_c = \frac{G_a}{\sum_i E_i X_i - H}$$

The economics of fire control have been discussed in [1]. If fixed costs are neglected as decision variables, the only cost factors to consider are,

L_i = the transportation cost per unit of the i^{th} force ($/man)

W_i = the cost per unit hour of the i^{th} force ($/man hr)

C = the cost of burn ($/acre).

An optimal decision requires the minimization of,

$$(3) \qquad C(Y_c - Y_a) + \sum_i L_i X_i + \frac{\sum_i W_i X_i G_a}{\sum_i E_i X_i - H}.$$

A simple adjustment transforms this nonlinear system into a parametric linear programming problem. If the cost of suppression is minimized for any given value of the burn cost, then the lowest value of burn cost plus suppression cost is the optimal solution.

If the total cost of burn is known, then $(Y_c - Y_a)$ is also known; hence,

$$\sum_i E_i X_i - H = \frac{G_a^2}{2(Y_c - Y_a)} = A$$

(4) $$\sum_i E_i X_i = A + H = B$$

(5) $$T_c = \frac{G_a}{A}$$

(6) $$\sum_i (L_i + W_i G_a / A) X_i + C(Y_c - Y_a) = \text{Minimum} .$$

III. Generalization of the Simultaneous Arrival Problem

In order to generalize the above problem, let

$$Y_c - Y_a = X_0$$

and

$$\sum_i E_i X_i - H = \alpha_0 + \sum_i \alpha_i X_i \text{ , a linear combination of the } X_i$$

for $(i = 1, \ldots, n)$. Then,

(4) $$X_0 \left[\alpha_0 + \sum_i \alpha_i X_i \right] = \frac{G_A^{\ 2}}{2}$$

(6) $$CX_0 + \sum_i L_i X_i + \sum_i D_i X_i \left[\alpha_0 + \sum_i \alpha_i X_i \right]^{-1} = Z \text{ (Min)}$$

where $D_i = W_i G_A$

(8) $$X_i \geq 0$$

We must minimize (6) subject to (4) and (8). If we assume $X_0 > 0$, then for

any particular value of X_0, say X_0^*,

$$(4a)\qquad \alpha_0 + \sum_i \alpha_i X_i = \frac{G_A 2}{2X_0^*} = R\,(X_0^*)$$

$$(6a)\qquad CX_0^* + \sum_i \overline{D}_i X_i = Z \text{ (Min.)}$$

where $\overline{D}_i = L_i + D_i/R(X_0^*)$,

$$(8)\qquad X_i \geq 0 \,.$$

Equations (4a), (6a), and (8) describe an elementary linear programming problem that is parametric simultaneously in the right-hand side as well as in the objective function.

The addition of other constraints on the X_i will not change the problem. Most generally we must solve,

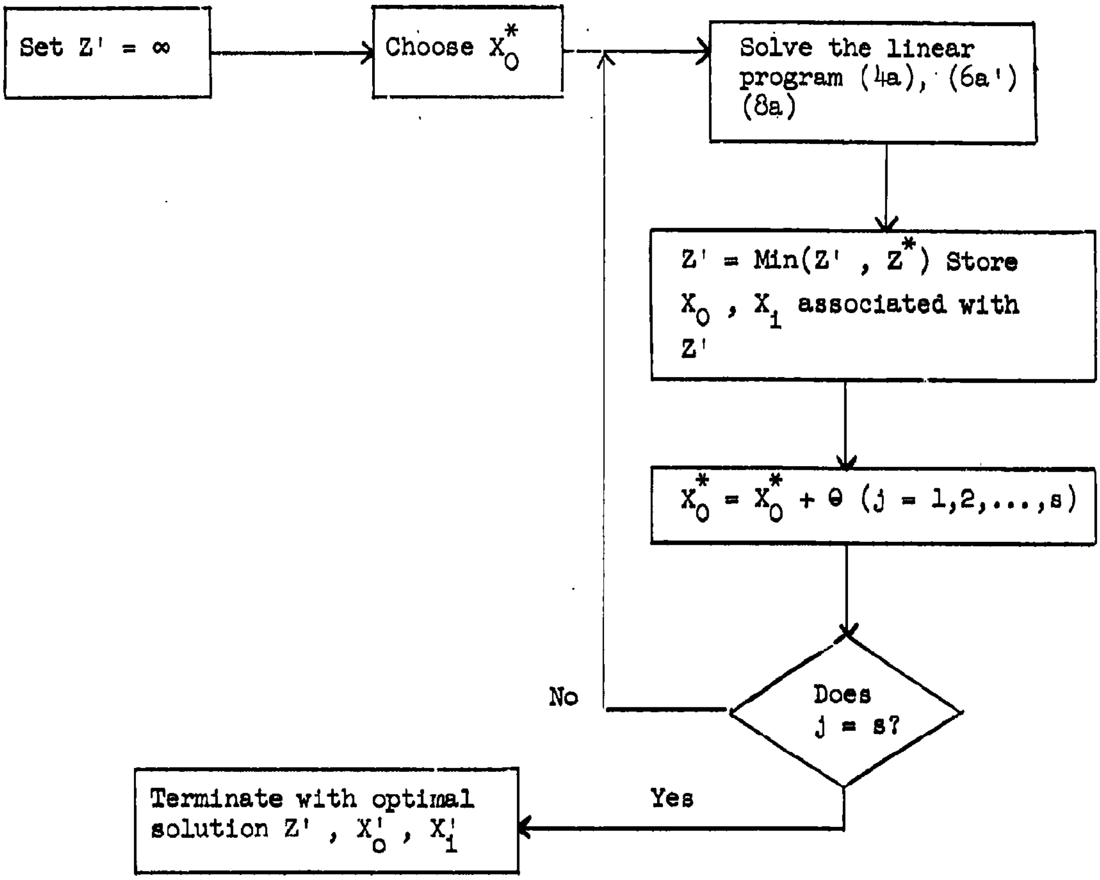
$$(4a)\qquad \alpha_0 + \sum_i \alpha_i X_i = R(X_0^*)$$

$$(8a)\qquad 0 \leq X_i \leq K_i \qquad\qquad i = 1,\ldots,n$$

$$(6a')\qquad \sum_i (L_i + D_i R(X_0^*)^P)X_i = Z \text{ (Min.)} \qquad\qquad -\infty < P < \infty$$

IV. Solution Algorithm for Simultaneous Arrival Problem

The following procedure yields the required optimal solution:



EXAMPLE: Simultaneous Arrival

In order to illustrate the above technique, consider the following example for two suppression crews that can arrive simultaneously at a fire.

$E_1 = 4$	$E_2 = 2$
$L_1 = 25$	$L_2 = 10$
$W_1 = 10$	$W_2 = 6$
$H = 20$	$T_a = 2$

For simplicity assume the fire is detected immediately after it starts. Then,

$$G_a = HT_a = 40 \text{ acres per hr.}$$

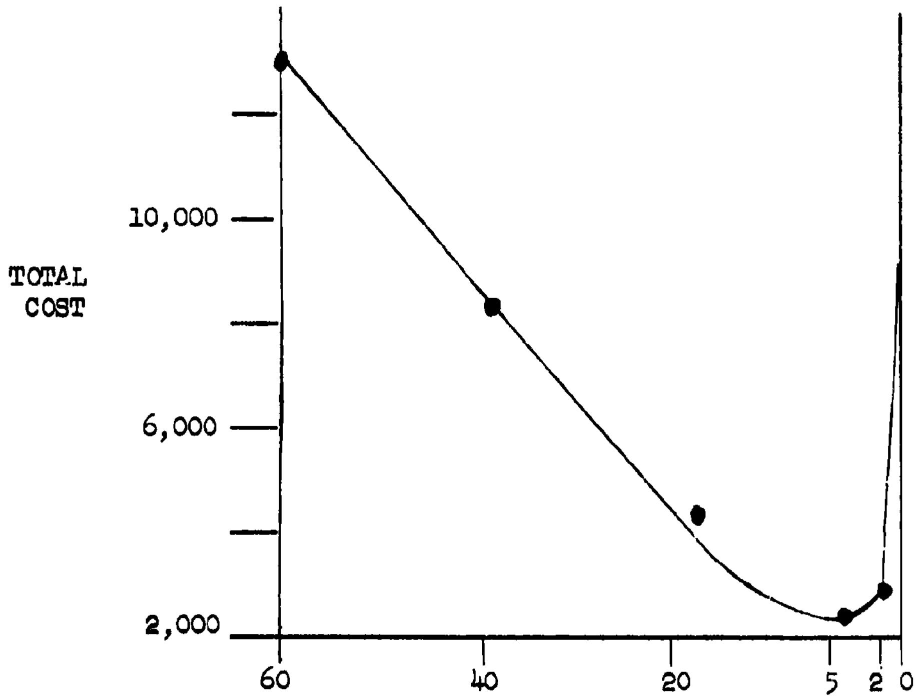
$$Y_a = \frac{1}{2}HT_a^2 = 40 \text{ acres .}$$

For C equal to $200 per acre, the following results were obtained:

TABLE I

X_0 acres	X_1 No. of forces	X_2 No. of forces	C_{supp} $	C_{burn} $	C_{total} $
60	$8\frac{1}{3}$	0	458	12,000	12,458
40	0	20	440	8,000	8,440
20	0	30	480	4,000	4,480
5	0	90	1,035	1,000	2,035
2	0	120	2,226	400	2,626

Notice that the optimal solution occurs where the cost of burn after arrival of crews is about equal to the cost of suppression. This type of solution agrees with the intuitive idea that it is uneconomic to spend more money per acre on suppression than the value per acre of the burning land. The results are plotted below in Figure 1.



ACRES BURNED

FIG. 1

A further analysis of the optimal solution yields some useful results. The minimum cost occurs at $X_0 = 5$ acres. The equations of this system are,

$$4X_1 + 2X_2 = 180$$

$$27.5X_1 + 11.5X_2 = Z(\text{Min}) .$$

The optimal solution is $X_2 = 90$, and $Z = \$1,035$. Suppose we now consider the use of a bulldozer for which

$$E_{bu} = 50$$

$$W_{bu} = \$25 .$$

What is the maximum transportation cost for which the bulldozer can be used? The transportation cost is important in the consideration of bulldozer usage because we must consider the cost of getting the bulldozer to the fire in the same time that the other forces require. This type of problem is easily analyzed by the dual linear programming problem.

V. Use of the Dual Linear Program

For every primal linear programming problem.

$$Ax = b$$

$$Cx = \text{Min} . \qquad x \geq 0$$

there is an associated dual,

$$yA \leq C$$

$$yb = \text{Max}. \qquad y \text{ unrestricted}$$

such that at optimality

$$yb = Cx$$

Furthermore, by the theorem of complementary slackness,

$$X_i = 0 \text{ whenever } ya_i < c_i$$

and

$$ya_i = c_i \quad \text{whenever} \quad X_i > 0 \ .$$

For a full treatment of the subject, see Chapters 1 and 4 of Gale [2].

Let the number of bulldozers used by represented by X_{bu}. From the above, if

$ya_{bu} < c_{bu}$, then $X_{bu} = 0$ and hence the optimal solution does not change.

For the optimal solution of Example 1, the dual is,

$$4Y_1 \leq 27.5$$

$$2Y_1 \leq 11.5$$

By the complementary slackness requirements,

$$2Y_1 = 11.5$$

$$Y_1 = 5.75$$

The dual is optimal because,

$$Y_1 b = 5.75(180) = 1035 = Cx \ .$$

The bulldozer should not be used if,

$$50Y_1 < (L_{bu} + \tfrac{1}{4}W_{bu})$$

or if,

$$287.5 < (L_{bu} + 6.25) \ .$$

If $L_{bu} > \$ 281.25$, the bulldozer should not be used. If the inequality is not satisfied, then another cycle of the simplex method will determine the correct number of bulldozers to use. Further, if they are used at the optimal acreage, the entire problem should be re-worked with the bulldozers considered in case there is a new optimal acreage allowance. Even for the case where bulldozers are not used at the optimum acreage, X_0^*, part of the problem must be re-solved. At some larger value of X_0, it is possible that bulldozers will be economical to use. Therefore, one should rework the problem for all $X_0 > X_0^*$. In the neighborhood of X_0^*, where the set of basic variables does not change, one can compute a region in which bulldozers are not used, thereby reducing the actual amount of re-computation of the linear program.

Note that the solution has not been restricted to integers. In practice a fraction of a man cannot be sent to a fire nor can you send a fraction of a bulldozer. Rather than complicate the linear programming problem with integer requirements one can arbitrarily send an additional man for each fraction greater than 0.3. Parks [1] has shown that it is cheaper to overkill a fire than to underman the suppression force. Therefore, little is to be gained by formulating complex integer programming problems. For problems with more constraints, one might also investigate fractional programming algorithms.

VI. Uncertain Growth Rates

The extension of Parks' model for uncertain growth rates follows directly from the argument in [1]. In actual practice G_a may only be measurable to a distribution function with mean μ and variance σ^2.

Taking the expected value of (6),

$$\mathcal{E}\left[\sum_i L_i X_i + \sum_i \frac{W_i G_a X_i}{A}\right] = \sum_i \mathcal{E}(L_i X_i) + \sum_i \mathcal{E}\frac{W_i G_a X_i}{A}$$

$$= \sum_i (L_i X_i) + \sum_i \frac{W_i X_i 2X_0}{\mathcal{E}(G_a)}$$

because

$$\frac{G_a}{A} = 2X_0/G_a$$

$$A = f(G_a) .$$

Therefore,

$$\mathcal{E}(A) = \frac{\mathcal{E}(G_a^2)}{2X_0}$$

and

$$\mathcal{E}(G_a^2) = (K^2 + 1)(\mathcal{E}(G_a))^2$$

where

$$K^2 = \frac{\sigma^2}{\mu^2} .$$

Then as before,

$$\text{(4a)} \qquad \sum_i \alpha_i X_i = \frac{(K^2 + 1)(\mathcal{E}(G_a))^2}{2X_0} + H$$

$$\text{(6b)} \qquad \sum \left(L_i + \frac{2X_0 W_i}{\mathcal{E}(G_a)}\right) X_i = \text{Minimum} .$$

The insertion of uncertainty into the problem will increase the number of men sent on the initial attack. The solution method here is the same as that outlined in Model 1.

VII. MODEL 2 Nonsimultaneous Arrival

The usual deployment situation is characterized by different arrival times for groups of forces dispatched at the same time. For example, a helitac crew should arrive early than a tanker truck crew under average conditions. Before developing a mathematical model of this situation note that values E for each force are important in eliminating a class of problems. For those cases where the most effective force is closest to the fire one shouldusually send as many of these forces as possible. Only under an unusual cost spread between types of forces would there be a change from this strategy.

Under most circumstances there areusually only a few types of forces that are sent initially. Let us consider just two types of forces and develop the mathematical structure of the model.

Let $G_1 = G_d + HT_{da}$ = growth rate at the time of attack of the first crew

T_i = inter-arrival time between crews

T_2 = time from attack to control for crew 2

T_1 = time from attack to control for crew 1 (the first to arrive)

E_i = effectiveness of the i^{th} crew.

T_{dA}= time from detection to arrival of the first crew

Using the time of attack of the first crew as a base, the growth rate of the fire is described by,

$$(9) \qquad \frac{dy}{dt} = G_1 - (E_1X_1 - H)\,T_i - (E_1X_1 + E_2X_2 - H)\,(t - T_i)\,.$$

Equation (9) holds for all non-negative values of X_1 and X_2. We must minimize,

$$(10) \qquad (L_1 + W_1T_1)\,X_1 + (L_2 + W_2T_2)X_2 + CX_0 \;.$$

Such a process is not trivial because T_1 and T_2 depend directly on X_1 and X_2 . The variables, X_0 , X_1 , and X_2 are related by,

$$(11) \qquad X_0 = \left[G_1 - \frac{(E_1X_1 - H)T_i}{2}\right] T_i + \frac{\left[G_1 - (E_1X_1 - H)T_i\right]^2}{2(E_1X_1 + E_2X_2 - H)}$$

The control times are given by,

$$(12) \qquad T_2 = \frac{G_1 - (E_1X_1 - H)T_i}{(E_1X_1 + E_2X_2 - H)} \quad ; \quad T_1 = T_i + T_2$$

At the boundary, however, we get different expressions for control times. For $X_2 = 0$, and $X_1 > 0$

$$(12a) \qquad T_1 = \frac{G_1}{E_1X_1 - H}$$

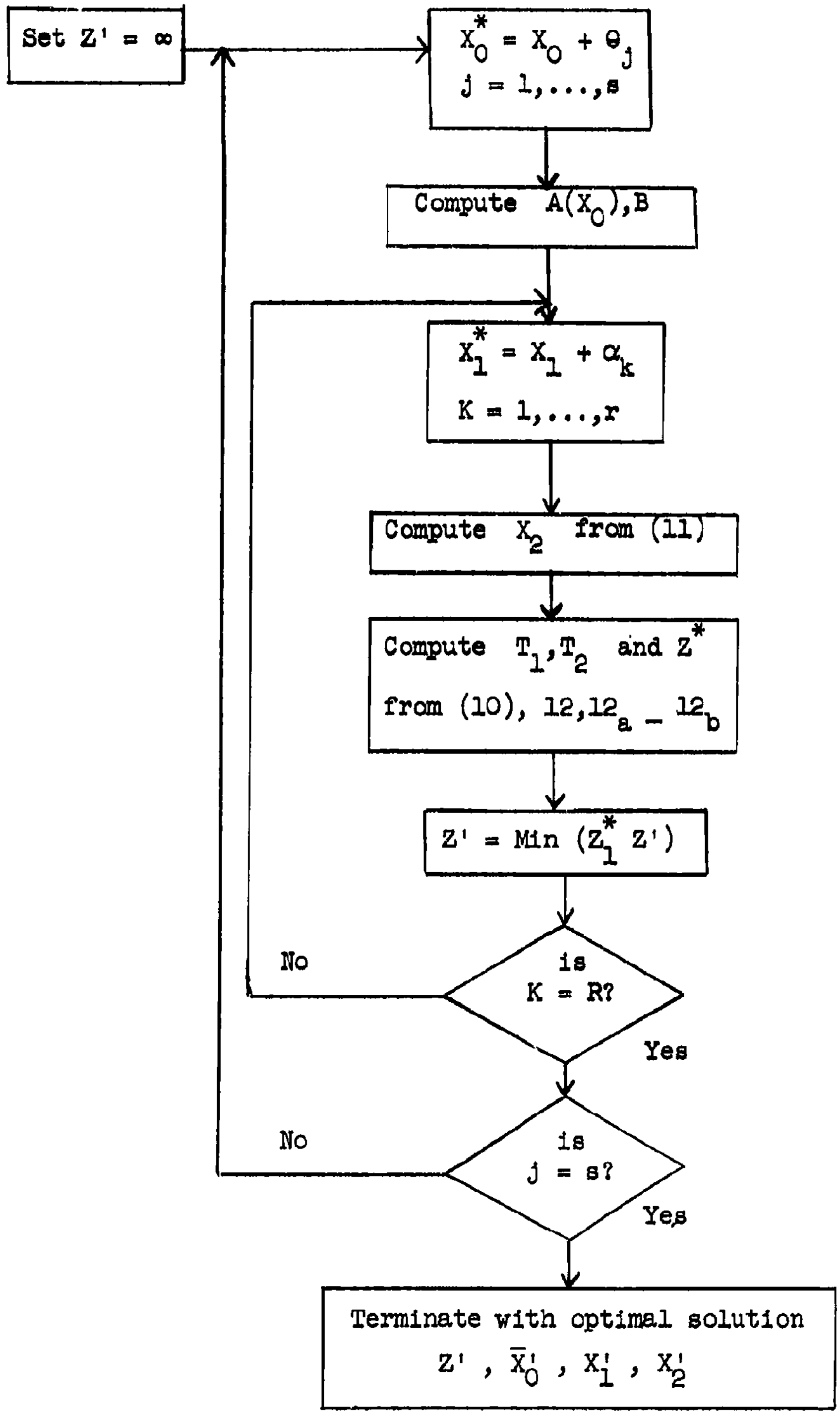
and for $X_1 = 0$, and $X_2 > 0$,

$$(12b) \qquad T_2 = \frac{G_1 + HT_i}{E_2X_2 - H} \;.$$

The variable X_1 is also bounded above and below for X_2 positive; its minimum value must not allow more than X_0 acres to burn before the second crew arrives, and its maximum value must not allow control of the fire before the second crew arrives. Thus, for $X_2 > 0$

$$\frac{G_1^2}{2E_1X_0} + \frac{H}{E_1} \leq X_1 \leq \frac{G_1}{E_1T_i} + \frac{H}{E_1} \quad \text{or} \quad A(X_0) \leq X_1 \leq B$$

The following algorithm will solve Model 2:



VIII. Generalization of Nonsimultaneous Arrival

This problem can be generalized to

(10a) $$Z = \mathrm{Min}\, f(X_0, X_1, X_2)$$

where

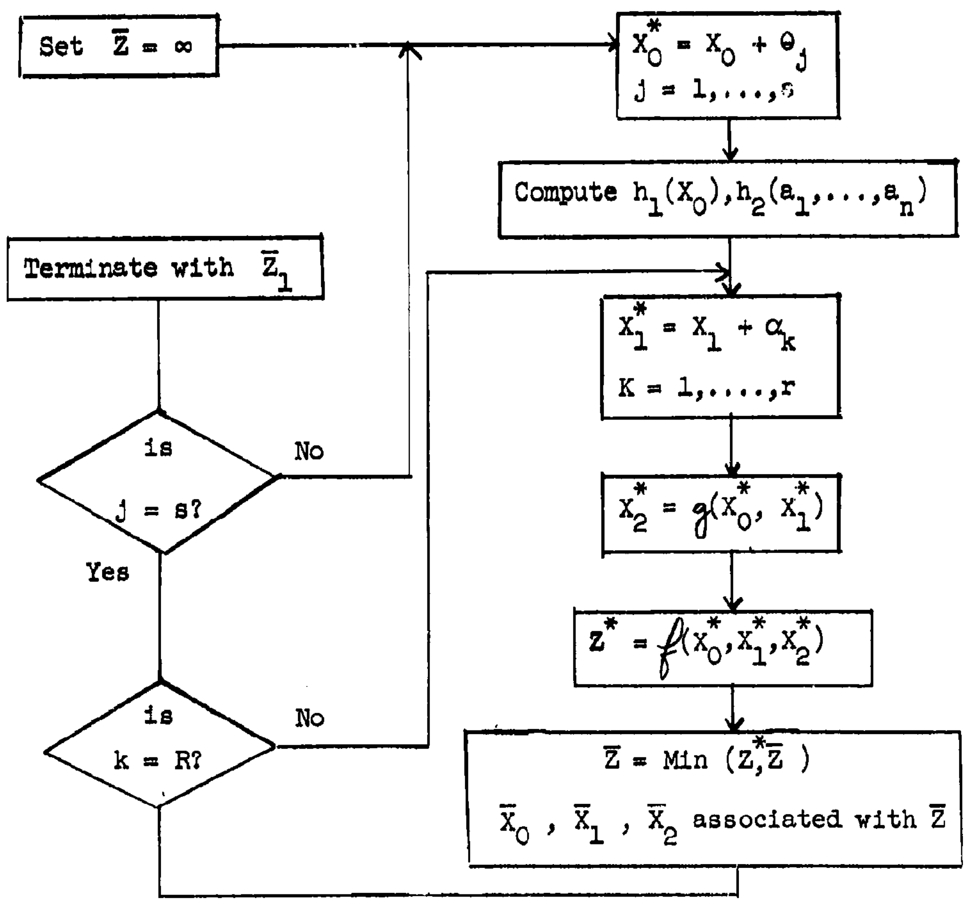
(11a) $$X_2 = g(X_0, X_1)$$

and

(13) $$h_2(X_0) \leq X_1 \leq h_1(a_1, \ldots, a_n)$$ (a_i are constants)

which is a dynamic programming problem.

The minimzation process can be computed using the following scheme:



IX. Use of the Planning Model

An optimization process for a system such as (10)-(13) is tedious and is not intended for use on individual fires. The process lends itself to solution by an iterative computer program. Such a program was written for the IBM 1620 located at the Survey Research Center at the University of California, Berkeley. From the solution of many possible cases using the same set of forces, a family of cost curves can be drawn; then depending on the value of H, G and E for each force, a dispatcher can easily choose the best mix of suppression forces for most fires. If the estimates of H are too low then it will be necessary to employ reinforcements. As Parks has shown [1], the reinforcement problem is similar to the initial attack problem and hence presents no great difficulties.

Before using the model to solve a specific problem, consider intuitively the results that we should obtain. For all values of X_0 less than X_0^*, the closer forces will begin to be used until at some point the control time is less than the inter-arrival time and only the earliest arriving forces can be used. For $X_1 = 0$, the equation for X_0 implies,

$$X_0 \geq (G_1 + HT_1/2)T_1 .$$

The critical value of X_0 is reached where

$$X_0^* = (G_1 + HT_1/2)T_1 \text{, the area burned until crew 2 arrives.}$$

If the most effective force does not arrive first, one would expect to use it only when the values of X_0 are large. If the number of early arriving forces is limited, then so is the minimum acreage that can be controlled. Thus there will be three critical regions to define and they are all linearly dependent on H. For large values of H the transitions should occur at higher values of X_0 than for smaller values of H.

EXAMPLE: Nonsimultaneous Arrival

In order to illustrate the use of the planning model consider the following problem: Three crews are available for initial attack on fires in a particular region of a forest. They are, (1) a 2-man helitac crew, (2) a 4-man tanker crew and (3) a small bulldozer. For values of H of 3, 10, and 50 acres per hr^2, find the optimal strategy.

Crew	Inter-arrival Time*	E	L_1	W_1
1	—	4	50	8
2	2/3	8	25	32
3	2/3	25	100	28

*Inter-arrival time from Crew 1 to other crews.

Let C = $200 per acre

G_1 = 5 acres per hour.

The two crew model can be solved for Crews 1 and 2 and for Crews 1 and 3; the lower envelope of cost curves will indicate the optimal policy among all three crews.

Some of the results of the computer solution are listed in Appendix A. There are 2 columns in the appendix that bear further explanation. MVS refers to the values saved in controlling the fire at the next lowest X_0 value. The value saved at $X_0 = 20$ acres compared to $X_0 = 30$ acres is $2,000 because MVS equals $C_{burn} \Delta X_0$. MC is the extra cost of suppression associated with controlling the fire at the lower X_0 value. These columns indicate the following requirement for an optimal solution:

$$X_0^* \text{ occurs where } MC = MSV . \quad (X_0^* \text{ is optimal } X_0)$$

Optimality then, is not defined as the X_0 value for which burn cost per

acre equals suppression cost per acre; such a condition may be true at lower values of H, but it is not true in general. Notice that at high values of H, the suppression effort is rather costly on a per unit acre basis; such a measure of spending efficiency is biased, however, because it does not reflect the potential acreage saved.

Returning to the simultaneous arrival case, notice that the marginal criterion of optimality holds there too. This concept is similar to the determination of optimal output in a production system; optimal output occurs where the marginal cost of the next unit produced equals the marginal revenue derived from the unit.

The results for the fixed inter-arrival problem are plotted in Figures 2, 3, and 4.

The optimal solutions of this model clearly agree with our intuitive assumptions. The cost curves have a typical convex shape and a minimum point. There is an error in the curves, however, caused by the lack of restrictions on using integral crews. For example, the cost of transporting a helitac crew is $25 per man, but the cost of sending only a one man crew is $50 per man. Therefore for each fractional crew there is an added fixed cost of transportation not considered here. The results will not be changed markedly because these fixed costs will only cause jumps in the cost curves at several points rather than causing changes in the cost curves.

For the lower values of H, the cost of suppression decreases as X_0 decreases, and then increases again. This behavior is caused by the balancing of extra transportation costs with decreased hourly work costs.

With or without these refinements, the multiple crew model can be used to advantage in choosing among different types of forces.

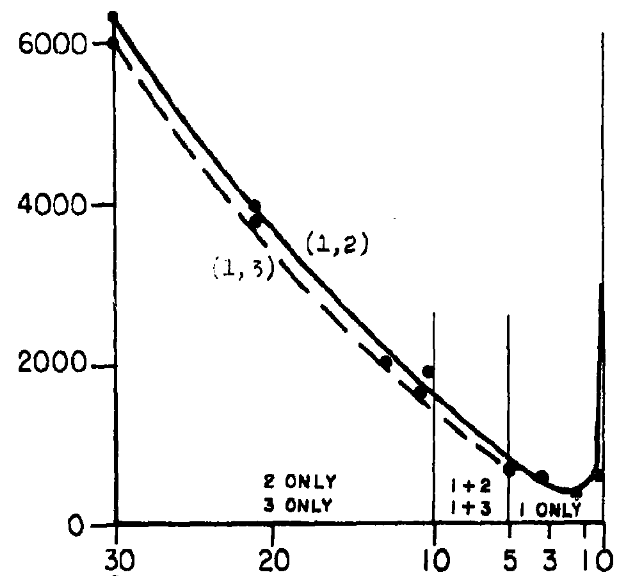


FIG. 2
H = 3

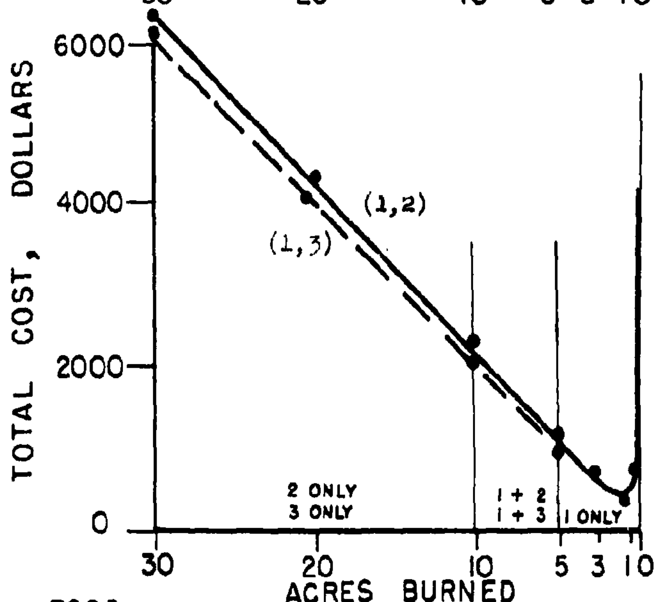


FIG. 3
H=10

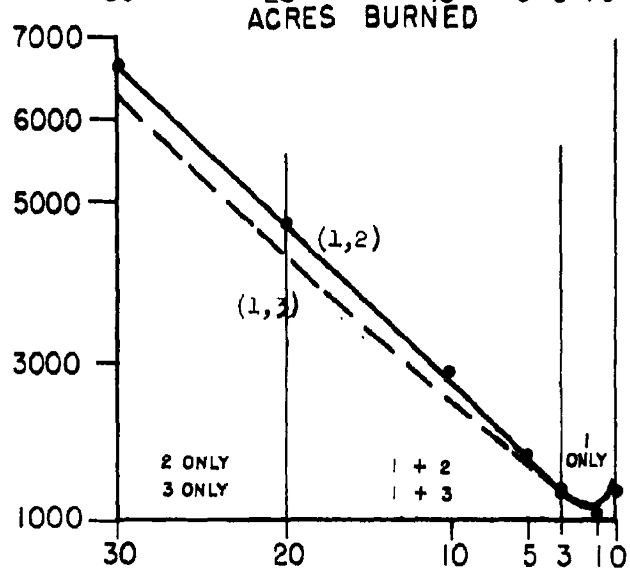
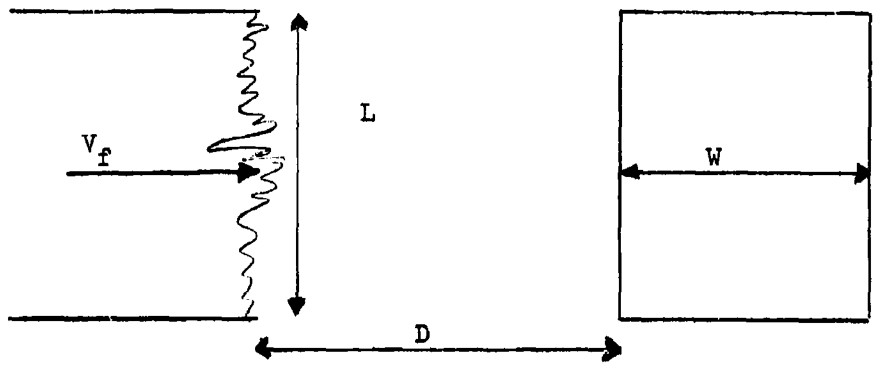


FIG. 4
H=50

X. Multiple Crews Fire Break Model

Jewell [3] has constructed the fire spread model illustrated in Figure 5.



V_f is the velocity of the fire front. L is the length of the front, D is a distance away from the front at time zero that a crew must move in order to build a fire break of width W by the time the fire front reaches the crew.

$$\alpha x/W = \text{feet of fire break constructed by } X \text{ men per hour.}$$

If the time to construct the break equals the travel time of the fire front,

$$T = LW/\alpha X \ .$$

The area burned = $A = V_f LT = L^2W(V_f/\alpha X)$. For the multiple crew case,

$$A = \frac{L^2WV_f}{\sum_i \alpha_i X_i}$$

Let $\alpha_i/W = \beta_i$ = the line building rate of the i^{th} force at a standard width W .

$$A = \frac{L^2V_f}{\sum_i \beta_i X_i} \qquad\qquad T = L/\Sigma\beta_i X_i$$

If A is held constant, then

$$(13) \qquad \sum_i \beta_i X_i = L^2 V_f / A$$

and

$$(14) \qquad \sum_i C_i X_i A / L V_f = \text{Minimum (where } C_i \text{ is the unit cost per man hour)}$$

forms a linear programming system whose solution yields the optimal mix of forces. An iterative procedure on A will define the overall optimal strategy given the unit cost of burn.

Now suppose that a fire spreads in N directions and N such fire breaks must be constructed. Because of the terrain and fuel type, let the area A_i, be fixed. Then,

$$(15) \qquad \sum_{i=1}^{N} \beta_{ij} X_{ij} = V_{f_j} L_j^2 / A_j \qquad \text{for all } j = 1,2,\ldots,M,$$

where β_{ij} = rate for the i^{th} force on the j^{th} fire front and

X_{ij} = number of the i^{th} type of force sent to the j^{th} fire front,

and

$$(16) \qquad \sum_{j=1}^{M} X_{ij} \leq K_i \qquad \text{for all } i$$

$$(17) \qquad \sum_i C_i \sum_j X_{ij} A_j / L_j V_{f_j} = \text{Minimum}$$

describes a linear programming problem. We can solve these problems with the same parametric procedure outlined in Model 1. The optimal solution of this system is given the best deployment strategy for all forces. On a campaign fire, where there are a number of divisions and sectors, such a model would be a useful tool to a fire boss; the linear programming device

automatically investigates the trade offs among costs of crews, line building ability and values burned.

As models are constructed that analyze other types of fire line strategies similar extensions to multivariable case can be readily made. In conclusion, I would suggest that extensions of these models are easily attained through the use of parametric programs that are solvable on a computer.

APPENDIX A

Fixed Inter-Arrival Time Problem

H	X_0	Crew 1	Crew 2	C_{total}	C_{supp}	MVS	MC
3	30	0	.49	$6,125	$ 125	$2,000	$ -28
3	20	0	.56	4,097	97	2,000	-26
3	10	0	.88	2,071	71	1,000	17
3	5	.91	.63	1,088	88	400	19
3	3	1.79	0	707	107	400	99
3	1	3.87	0	406	206	150	467
3	.25	13.24	0	723	673		
10	30	0	1.59	6,254	254	2,000	-62
10	20	0	1.83	4,192	192	2,000	-36
10	10	0	3.16	2,156	156	1,000	40
10	5	2.29	1.40	1,196	196	400	15
10	3	3.54	0	811	211	400	88
10	1	5.62	0	499	299	150	463
10	.25	14.99	0	812	762		
50	30	0	12.55	6,620	620	2,000	63
50	20	4.21	9.06	4,683	683	2,000	95
50	10	10.20	2.81	2,778	778	1,000	14
50	5	12.29	1.40	1,792	792	400	4
50	3	13.31	.84	1,396	796	400	35
50	1	15.62	0	1,031	831	150	339
50	.25	24.99	0	1,320	1,270		

KEY

H	Acceleration
X_0	Acres burned after arrival of first crew
MVS	Value of burned acres saved in reducing X_0 at each step
MC	Additional cost of suppression in reducing X_0 at each step

REFERENCES

[1]. Parks, G. M., and W. S. Jewell, "A Preliminary Model for Initial Attack," Research Report F-1, Operations Research Center, University of California, Berkeley (15 June 1962).

[2]. Gale, D., Theory of Linear Economic Models, McGraw-Hill Book Company, 1961.

[3]. Jewell, W. S., "Forest Fire Problems--A Progress Report," Research Report F-2, Operations Research Center, University of California, Berkeley (15 June 1962).

[4]. McMasters, A. W., "Preliminary Analysis of the Influence of Hand Crews on Fire Growth," Research Report ORC 63-7, Operations Research Center, University of California, Berkeley, (1 May 1963).